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Calculating the Probability of Successfully Executing the Kill Chain to Analyze Hypersonics

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Introduction

Several years ago, the Air Force established the detailed concept of the kill chain as "find-fix-track-target-engage-assess" or F2T2EA. The systems required for successfully executing the kill chain are an integration of C4ISR and "kill-or-disable" systems. The recent development and testing of hypersonic technology by our national security community as well as our adversaries, notably China and Russia, have heightened the need for analytic capabilities. The threat is highlighted by the fact that a weapon traveling at Mach 5 can travel approximately 625 miles in less than 10 minutes. The requisite analytic capabilities are crucial for requirements definition, design, analyses of alternatives (AoAs), cost and operational performance trade studies, and development of operational tactics and strategies. The F2T2EA kill chain can be applied to air and missile *defense* systems defending against enemy hypersonic weapons or applied to *offensive* hypersonic weapons attacking enemy ground targets. In trade-space studies, analysts endeavor to determine how we should invest in combinations of new ISR capabilities, new command and control capabilities, and new hypersonic munition capabilities.

A recent publication in *Real Clear Defense* (Frasier et al., 2020) highlighted some of the challenges presented by hypersonic capabilities:

> "There is a pathway to overcome these challenges. Just like any other missile defense system, there must be an effective chain of weapon engagement capabilities to 'Find, Fix, Track, Target, Engage, and Assess' (F2T2EA) hypersonic vehicles throughout their flight profiles. Foundational to the kill chain is the ability to Find, Fix, Track and Target (F2T2). Developing a weapons-quality target track is necessary for any interceptor to engage and destroy the target. First things first, the defense design must get the F2T2 system right, or even the best interceptor in the world will be unable to engage the target."

The June 2007 *Phalanx* contained my article, "Quantifying 'Persistence' in the Context of Find-Fix-Finish (FFF)" (Rice, 2007) which focused more generally on a three-step kill

In 2016, The Mitchell Institute (Hallion et al., 2016) determined:

"One of the key advantages hypersonic weapons will leverage is their potential to compress the time it takes for a weapon to travel to its target, redefining engagement opportunities and allowing more operational flexibility. The reduction of so-called 'shooter-to-target' time in an age of increasingly more lethal and mobile weapons is critical to preserving military power. US efforts to perform targeted strikes on terrorist groups and their leadership have embodied the challenge of weapon transit time in the recent past, as these groups have rarely used fixed bases of operation and often vary the location and duration of their meetings in order to complicate targeting efforts.

By increasing the utility of intelligence to build actionable targets, military commanders would have a more diverse pallet of targeting options, enabled by hypersonics. Locating, engaging, closing with, and destroying the opponent has always defined successful military operations. In modern campaigns, this unfolds as the find, fix, target, track, engage, and assess process—or the 'F2T2EA chain.' Advanced ISR and C2 systems have evolved and improved the US military's ability to find, fix, target, and track a host of potential aim points. But the tools of engagement—gravity bombs, short-range standoff missiles, and subsonic cruise missiles, among others—are becoming less decisive as more opponents begin to field modernized versions of these weapons themselves. With its military advantage eroding, the US' command flexibility and freedom of action in operations is increasingly affected. Hypersonic weapons would reverse this trend. A commander who possesses hypersonic strike capability can choose to exercise a wider range of options, with the speed of hypersonics allowing more reflective decision-making, instead of reflexive decisions driven by tight windows of opportunity."

chain referred to as FFF, with the understanding that the mathematics could be extended to a more robust kill chain with more steps and therefore more resolution. Recently, I have been approached by several analysts and agencies to mathematically expand FFF to the established five-step (F2T2E) kill chain. The current work borrows extensively from this previous article for derivations and methodology. The purpose of this article is to develop an equation for the probability of successfully executing the five-step F2T2E kill chain. This calculation will enable quick analyses and complement large, more complex simulation models. Analysts can use it to analyze various trade-spaces such as systems, tactics, operations, and costs in terms of their impact on F2T2E. The intent is NOT to minimize the final sixth step of "assessing" in the kill chain, but rather to develop the analytics for the first five steps through the "engage" step with a later emphasis on the probability of successful kill/disable of the target.

The first task is to derive an equation for the probability of successfully executing the kill chain, $P_{F2T2E}(T)$ in time T. The next step is to derive the partial derivatives of the probability equation with respect to the input variables to analyze the impact of each variable on improvements to the overall probability of success. And, finally, I show some results and insights.

Problem Setup and Solution

$P_{F2T2E}(T)$ is basically a sequence of conditional probabilities delineating the kill chain backward:

P[engage | target] * *P*[target | track] * *P*[track | fix] * *P*[fix | find] * *P*[find].

This is a reasonable approach when the specific time windows for the individual event probabilities are known. However, this is seldom the case. Most often, we are faced with a given time window, or window of target opportunity, in which the entire F2T2E process must take place. For example, our window of target opportunity may be from the time a hypersonic missile is launched until it impacts a target or the time a terrorist drives to and enters a restaurant until the time he departs the restaurant. We must find, fix, track, target, and engage within that "shooter-to-target" time window.

Graphically, this time window, T, is shown in Figure 1 with the sequential individual F2T2E event times.

Once we find, we can fix; once we fix, we can track; once we track, we can target; and once we target, we can

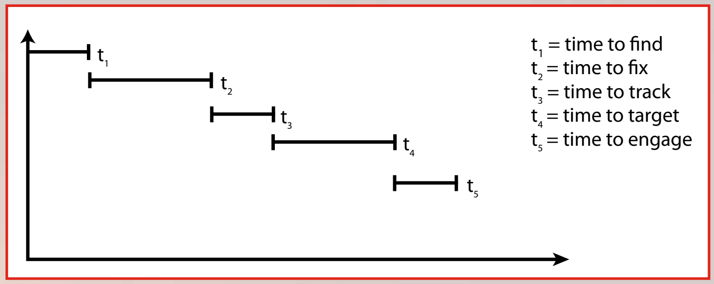

Figure 1. Time window, T, and times to find, fix, track, target, and engage.

engage. So, instead of conditional probabilities with known times for each event, we want to solve for the probability of accomplishing all the events in the window of opportunity, T, which is defined as:

$P_{F2T2E}(T) =$

$$P[(t_1 \leq T) \cap (t_2 \leq T - t_1) \cap (t_3 \leq T - t_1 - t_2) \cap (t_4 \leq T - t_1 - t_2 - t_3) \cap (t_5 \leq T - t_1 - t_2 - t_3 - t_4)]. \quad (1)$$

Koopman (1946, p. 19) states that "when the looking is done continuously during a time t under unchanging conditions, the probability $p(t)$ of detection is given by":

$$p(t) = 1 - e^{-\lambda t}, \quad (2)$$

where λ is the rate of "detection." Since this is a cumulative probability, the time between detections follows the exponential density function:

$$f(t) = \lambda e^{-\lambda t}. \quad (3)$$

What Koopman describes is formalized by Ross (1983, p. 31) in that these assumptions of a counting process and of stationary independent increments, "the distribution of the number of events that occur in any interval of time depends only on the length of the time interval," make these Poisson processes, which have been used extensively in stochastic modeling.

This approach applies Koopman's methodology pertaining to detection and assumes that this is representative of each of the five events, find-fix-track-target-engage. While analysts have traditionally modeled the probability of damage given attacks, this approach incorporates the probability of "engage" within a time interval. As in the previous approach, considering this methodology of using time windows is more appropriate for mobile targets that require "finding and fixing" since these targets are vulnerable to strikes once located. Representing the time for "engage" events as following an exponential distribution is appropriate for cases where strike assets are

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loitering to flex on targets of opportunity as they appear dispersed across a region. This is supported by assuming that "find, fix, track, target" means that the target is kept on "track" or that the target doesn't break track.

As in my 2007 article, the λ_is encompass all the spatial aspects of target density, revisits, and geography to enable mathematical inclusion of all these issues in the temporal domain. So, allowing that the λ_is (λ_1 = "find" rate, λ_2 = "fix" rate, etc.) for each of the events (F2T2E) may not be equal (it's unlikely they would be equal and that solution is trivial) the solution to Equation (1) becomes:

$P_{\mathrm{F2T2E}}(T)$ =

$P[(t_1 \le T) \cap (t_2 \le T - t_1) \cap (t_3 \le T - t_1 - t_2) \cap (t_4 \le T - t_1 - t_2 - t_3) \cap (t_5 \le T - t_1 - t_2 - t_3 - t_4)]$ =

$$\int_0^{T} \lambda_1 e^{-\lambda_1 \tau_1} \int_0^{T-t_1} \lambda_2 e^{-\lambda_2 \tau_2} \int_0^{T-t_1-t_2} \lambda_3 e^{-\lambda_3 \tau_3} \int_0^{T-t_1-t_2-t_3} \lambda_4 e^{-\lambda_4 \tau_4} \int_0^{T-t_1-t_2-t_3-t_4} \lambda_5 e^{-\lambda_5 \tau_5}\, d\tau_5 d\tau_4 d\tau_3 d\tau_2 d\tau_1 \qquad (4)$$

Integrating the right-hand side of Equation (4) yields the following:

$$\begin{aligned} P_{\mathrm{F2T2E}}(T) = 1 - {} & e^{-\lambda_1 T}\left(\frac{\lambda_2\lambda_3\lambda_4\lambda_5}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)(\lambda_1-\lambda_5)}\right) \\ & -e^{-\lambda_2 T}\left(\frac{\lambda_1\lambda_3\lambda_4\lambda_5}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)(\lambda_2-\lambda_5)}\right) \\ & -e^{-\lambda_3 T}\left(\frac{\lambda_1\lambda_2\lambda_4\lambda_5}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)(\lambda_3-\lambda_5)}\right) \\ & -e^{-\lambda_4 T}\left(\frac{\lambda_1\lambda_2\lambda_3\lambda_5}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)(\lambda_4-\lambda_5)}\right) \\ & -e^{-\lambda_5 T}\left(\frac{\lambda_1\lambda_2\lambda_3\lambda_4}{(\lambda_5-\lambda_1)(\lambda_5-\lambda_2)(\lambda_5-\lambda_3)(\lambda_5-\lambda_4)}\right). \end{aligned} \qquad (5)$$

The general equation for this P_{F2T2E}(T), with i, j = 1, 2, . . ., 5, is:

$$P_{\mathrm{F2T2E}}(T) = 1 - \sum_i \frac{\left(e^{-\lambda_i T}\left(\prod_{i \ne j} \lambda_j\right)\right)}{\prod_{i \ne j}(\lambda_i - \lambda_j)}. \qquad (6)$$

Now, letting $\Theta_i = 1/\lambda_i$ be the mean-time-to event *i* (Θ_1 = mean-time-to "find," Θ_2 = mean-time-to "fix," etc.), Equation (5) becomes

$$\begin{aligned} P_{\mathrm{F2T2E}}(T) = 1 - {} & e^{-T/\theta_1}\left(\frac{\theta_1{}^4}{(\theta_2-\theta_1)(\theta_3-\theta_1)(\theta_4-\theta_1)(\theta_5-\theta_1)}\right) - \\ & e^{-T/\theta_2}\left(\frac{\theta_2{}^4}{(\theta_1-\theta_2)(\theta_3-\theta_2)(\theta_4-\theta_2)(\theta_5-\theta_2)}\right) - e^{-T/\theta_3}\left(\frac{\theta_3{}^4}{(\theta_1-\theta_3)(\theta_2-\theta_3)(\theta_4-\theta_3)(\theta_5-\theta_3)}\right) - \\ & e^{-T/\theta_4}\left(\frac{\theta_4{}^4}{(\theta_1-\theta_4)(\theta_2-\theta_4)(\theta_3-\theta_4)(\theta_5-\theta_4)}\right) - e^{-T/\theta_5}\left(\frac{\theta_5{}^4}{(\theta_1-\theta_5)(\theta_2-\theta_5)(\theta_3-\theta_5)(\theta_4-\theta_5)}\right). \end{aligned} \qquad (7)$$

The general equation for this $P_{\mathrm{F2T2E}}(T)$, with *i, j* = 1, 2,…, 5, is:

$$P_{\mathrm{F2T2E}}(T) = 1 - \sum_i \frac{\left(e^{-T/\theta_i} * (\theta_i)^4\right)}{\prod_{j \ne i}(\theta_j - \theta_i)}. \qquad (8)$$

Example

Consider an example with the shooter-to-target time window *T* varying from two minutes to 24 minutes while Θ_1 = 3 minutes, Θ_2= 2.5 minutes, Θ_3 = 2 minutes, Θ_4 = 1 minute, and Θ_5 = 0.5 minutes. Figure 2 shows $P_{\mathrm{F2T2E}}(T)$ (Equations (7) and (8)) as a function of the time window, *T*.

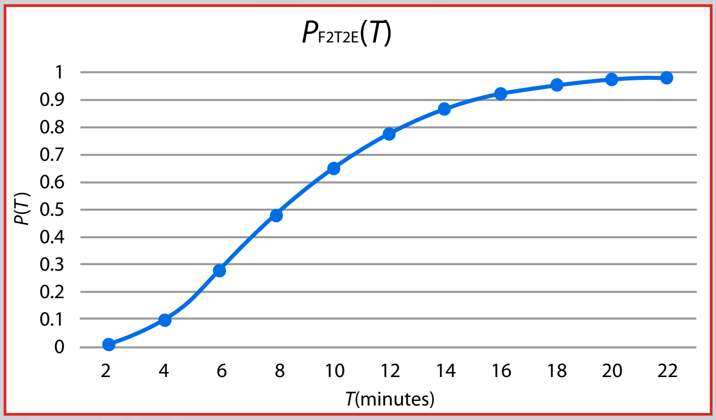

Figure 2. ***$P_{\mathrm{F2T2E}}(T)$ as a function of T.***

In this example, if Θ_1 = 3 minutes, Θ_2 = 2.5 minutes, Θ_3 = 2 minutes, Θ_4 = 1 minute, Θ_5 = 0.5 minutes, and we want a $P_{\mathrm{F2T2E}}(T) \ge 0.80$, we graphically solve Equation (7) (see Figure 2) for *T*, which yields *T* = 12.346 minutes; or, if the window of opportunity is equal to or greater than 12.346 minutes, then $P_{\mathrm{F2T2E}}(T) \ge 0.80$.

Impact of the Θ_is

Partial derivatives may be used to determine how the Θ_is impact the overall $P_{\mathrm{F2T2E}}(T)$, for notational purposes, *P*. These quantify the *changes in the $P_{\mathrm{F2T2E}}(T)$ given incremental changes in each Θ_i for a given T*. The partial derivatives, $\partial P/\partial \Theta_i$, with *i, j, k, l* =1, 2, 3, 4, 5 (i.e., the five events of F2T2E), are:

$$\frac{\partial P}{\partial \theta_i} = -e^{-T/\theta_i}\left[\frac{A^*}{\sum_{j\neq i}(\theta_j-\theta_i)^2} + \frac{T\,\theta_i^2}{\sum_{j\neq i}(\theta_j-\theta_i)}\right] - \sum_{j\neq i}\left(e^{-T/\theta_j}\left[\frac{-\theta_j^4}{(\theta_i-\theta_j)^2\prod_{k\neq j\neq i}(\theta_k-\theta_j)}\right]\right), \quad (9)$$

where

$$\boldsymbol{A}^* = 4\theta_i^3\left[\prod_{j\neq i}(\theta_j-\theta_i)\right] - \theta_i^4\left[4\theta_i^3 - 3\theta_i^2\left(\sum_{j\neq i}\theta_j\right) + 2\theta_i\left(\sum_{j\neq k\neq i}(\theta_j\theta_k)\right) - \left(\sum_{j\neq k\neq l\neq i}(\theta_j\theta_k\theta_l)\right)\right]. \quad (10)$$

These partial derivatives can now be used to calculate the impact of improvements or degradations in the individual events of F2T2E. Some calculated values of Equation (9) are shown in Table 1. Let Θ_1 = 3 minutes, Θ_2 = 2.5 minutes, Θ_3 = 2 minutes, Θ_4 = 1 minute, and Θ_5 = 0.5 minutes. The shaded cells in Table 1 identify the most negative values for each value of *T*, the importance of which will be discussed later.

Table 1. Partial derivatives as a function of *T* (minutes): Θ_1 = 3 minutes, Θ_2 = 2.5 minutes, Θ_3 = 2 minutes, Θ_4 = 1 minute, and Θ_5 = 0.5 minutes.

T	$\partial P/\partial \Theta_1$	$\partial P/\partial \Theta_2$	$\partial P/\partial \Theta_3$	$\partial P/\partial \Theta_4$	$\partial P/\partial \Theta_5$
2	−0.00283	−0.00332	−0.00400	−0.00676	−0.01020
4	−0.02490	−0.02835	−0.03286	−0.04740	−0.05900
6	−0.05725	−0.06323	−0.07033	−0.08783	−0.09628
8	−0.07832	−0.08375	−0.08934	−0.09829	−0.09903
10	−0.08177	−0.08453	−0.08649	−0.08527	−0.08113
12	−0.07249	−0.07238	−0.07107	−0.06377	−0.05830
14	−0.05768	−0.05557	−0.05241	−0.04338	−0.03854
16	−0.04252	−0.03950	−0.03583	−0.02767	−0.02407
18	−0.02963	−0.02652	−0.02317	−0.01687	−0.01444
20	−0.01977	−0.01705	−0.01436	−0.00994	−0.00840
22	−0.01276	−0.01059	−0.00862	−0.00571	−0.00478
24	−0.00801	−0.00640	−0.00504	−0.00321	−0.00267

Notice that Table 1 shows negative values for the partial derivatives, since decreases or improvements in the Θ_is result in increases or improvements in $P_{F2T2E}(T)$, and vice versa.

Equation (9) is extremely useful in determining where to most effectively improve the kill chain. It is interesting to note that if the Θs are simply reordered keeping the same values just in a different order, then the values of the partials will follow the reordering and keep their original values and $P_{F2T2E}(T)$ also remains the same. For example, let Θ_1 = 3, Θ_2 = 2.5, Θ_3 = 2, Θ_4 = 1.5, Θ_5 = 1, and *T* = 16. Then, reordering the Θs to be Θ_1 = 1, Θ_2 = 1.5, Θ_3 = 2, Θ_4 = 2.5, Θ_5 = 3, and keeping *T* = 16 yields the results in Table 2.

Partial Derivatives Vary over Time, T

There is now an equation for the probability of successfully executing the F2T2E kill chain, $P_{F2T2E}(T)$, as a function of the shooter-to-target time window (*T*) and the expected times to complete each of the five elements (Θ_i) of the F2T2E chain. Also, the analyst has equations for the partial derivatives of $P_{F2T2E}(T)$ to calculate the impact of improvements or degradations in the Θ_is. Next, let's examine how the partial derivatives from Equation (9) vary over *T*.

Table 1 includes highlighted cells in each row corresponding to a different value of *T*. These highlighted cells contain the largest magnitude or most negative partial derivative for a given value of *T*. This means that the specific Θ_i that produces the largest incremental change in $P_{F2T2E}(T)$ changes depending on the time window, *T*. This is shown in Figure 3. Note that the negative partial derivatives have been multiplied by –1 so that the plot is expressed in positive values for visual purposes.

Figure 3 displays the positive partial derivatives of $P_{F2T2E}(T)$ with respect to each of the Θ_is over varying time windows, T (2 – 24 minutes) using Θ_is set to Θ_1 = 3, Θ_2 = 2.5, Θ_3 = 2, Θ_4 = 1, Θ_5 = 0.5.

Figure 3 and Table 1 produce an interesting insight. When the time window *T* is relatively long such as 20 minutes, $\partial P/\partial \Theta_1$ is the most negative (positive in Figure 3 due to multiplication by –1) of the partial derivatives and $P_{F2T2E}(T)$ increases more rapidly by reducing Θ_1, the time it takes to "find" the target or reduce the largest Θ_i. But, at some point working backward from 20 minutes (between *T* = 8 and *T* = 10 minutes), $\partial P/\partial \Theta_5$ becomes the most negative (again, shown as positive in Figure 3) and it is more advantageous to decrease Θ_5 to get the largest improvement in $P_{F2T2E}(T)$; it makes most sense to improve

Table 2. Partial derivatives of *P*(*T*) with Θ_is reordered.

Θ_1s	$\partial P/\partial \Theta_1$	$\partial P/\partial \Theta_2$	$\partial P/\partial \Theta_3$	$\partial P/\partial \Theta_4$	$\partial P/\partial \Theta_5$	*P*(T) = *P*(16)
3, 2.5, 2, 1.5, 1	−0.0497	−0.0473	−0.0440	−0.0401	−0.0358	0.8912
1, 1.5, 2, 2.5, 3	−0.0358	−0.0401	−0.0440	−0.0473	−0.0497	0.8912

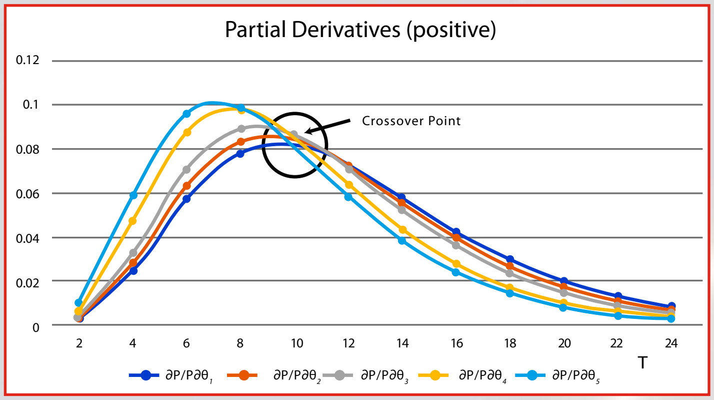

Figure 3. Positive partial derivatives as a function of *T*.

the fastest part of the kill chain (engage) rather than the slowest (find)—a somewhat counterintuitive result.

The salient insight is that for large values of *T*, reducing the *largest* Θ_i yields the largest increase in $P_{F2T2E}(T)$. But, as *T* gets smaller, reducing the *smallest* Θ_i yields the largest increase in $P_{F2T2E}(T)$. It all depends on the values of T and the Θ_is. The bottom line: it is not always the case that we should improve the slowest part of the kill chain to improve the overall probability of successfully executing the F2T2E kill chain.

Conclusion

This derived equation for $P_{F2T2E}(T)$ and its associated partial derivatives enable the analyst to create and evaluate trade spaces for tactical and operational performance capabilities to determine requirements, conduct design trades, perform cost-benefit studies and AoAs, and define areas where more detailed large-scale Monte Carlo simulation studies are needed. It should provide another arrow in the analyst's quiver to support decision making for advancing, or defeating, hypersonic weapon systems as well as other systems involved in the F2T2E kill chain.

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Dr. Roy E. Rice graduated from the U.S. Air Force Academy in 1975 with a BS in mathematics. Roy retired from the Air Force in 1995. In 2018, he retired as the chief engineer at Teledyne Brown Engineering in Huntsville, Alabama. Roy has been involved with MORS at all levels. He has been the chair and co-chair of the Reliability, Maintainability and Logistics working group, co-chair of the COEA working group, a composite group chair, tutorials coordinator for the 64th MORS Symposium, and he participated as a panelist for the Junior/Senior Analyst program at the 64th MORS Symposium. Roy has presented papers and led various working groups at many minisymposia and workshops and has also conducted tutorial presentations at several MORS Symposiums. He has also won the Barchi Prize twice and the Rist Prize once. Roy was elected to the MORS Board of Directors in 1996, was elected as Vice President for Meeting Operations in 1998–1999, and was MORS President in 2000–2001 and presided over the 69th MORS Symposiums. He was elected a Fellow of the Society in 2003. Roy was awarded the Vance R. Wanner Award in 2006 and the John K. Walker Award in 2008. In 2006, Roy was presented with the Air Force's Analyst Lifetime Achievement Award. In 2015, Roy was inducted by INFORMS as an Edelman Laureate.